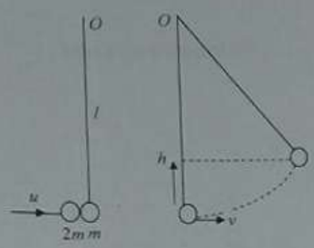
1. A particle of mass m hangs in equilibrium at one end of a light inextensible string of length l whose other end is tied to a fixed point O. Another particle of mass $2m$ collides horizontally with velocity u with the first particle and coalesces with it. Find the velocity with which the composite particle begins to move.
Show that if $u=\sqrt{gl}$, then the composite particle reaches a maximum height of $\frac{2l}{9}$ above its initial level.



Let v be the velocity with which the composite particle begins to move.

Apply $\underline{I}=\Delta(M\underline{v})$ for the system:

$\longrightarrow \quad 0=3mv-2m\times u$ (5)

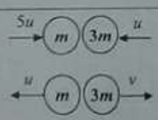
$\Rightarrow v=\frac{2u}{3}$. (5)

By the Conservation of Energy, $(3mg)h=\frac{1}{2}(3m)v^2$, where h is the required height. (10)

$\therefore \; h=\frac{v^2}{2g}=\frac{4u^2}{9(2g)}=\frac{4gl}{18g}=\frac{2l}{9}$. (5)

---

2. A particle P of mass m and a particle Q of mass $3m$ move on a smooth horizontal table along the same straight line towards each other with speeds $5u$ and u respectively, as shown in the figure. After their impact, P and Q move **away from each other** with speeds u and v respectively. Find v in terms of u, and show that the coefficient of restitution between P and Q is $\frac{1}{3}$.



Apply $\underline{I}=\Delta(M\underline{v})$ for the system:

$\longrightarrow \quad 0=(3mv-mu)-(5mu-3mu)$ (5)

$\Rightarrow 3mv=3mu$ (5)

$\Rightarrow v=u$. ------------(1)

By Newton's law of restitution: (10)

$v+u=e(5u+u)$

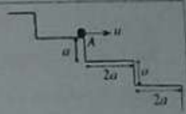
$(1)\Rightarrow 2u=6eu$

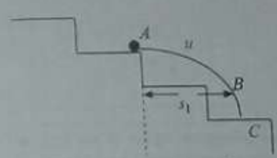
$\therefore \; e=\frac{1}{3}$. (5)

25

---

3. A particle P, projected horizontally with velocity u given by $u=\frac{3}{2}\sqrt{ga}$ from a point A at the edge of a step of a fixed stairway perpendicular to that edge, moves under gravity. Each step is of height a and length $2a$ (see the figure). Show that the particle P will not hit the first step below A, and it will hit the second step below A at a horizontal distance $3a$ from A.





For the motion of P, apply $s=ut+\frac{1}{2}at^2$;

from A to B: $a=\frac{1}{2}gt_1^2$, where t_1 is the time taken to reach the level of the 1st step below A. (5)

$\therefore t_1=\sqrt{\frac{2a}{g}}$.

Let s_1 be the horizontal distance moved in time t_1

$\longrightarrow$ from A to B: $s_1=u\times t_1=\frac{3}{2}\sqrt{ga}\times\sqrt{\frac{2a}{g}}=\frac{3}{\sqrt{2}}a>2a$. (5) (5)

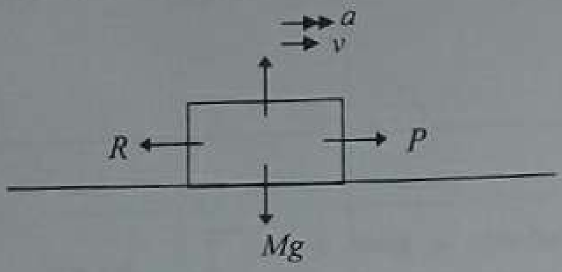
Thus P will not hit the 1st step below A.

Time taken from A to C is $t_2=\sqrt{\frac{2(2a)}{g}}$. (5)

$\longrightarrow \quad s=ut_2=\frac{3}{2}\sqrt{ga}\cdot 2\sqrt{\frac{a}{g}}=3a$. (5)

25

4. A car of mass M kg moves along a straight level road against a resistance of constant magnitude R N. At an instant when the car is moving at speed v m s^{-1}, its acceleration is a m s^{-2}. Show that the power of its engine at this instant is $(R + Ma)v$ W.
The car then moves with a constant speed v_1 m s^{-1} against a resistance of the same constant magnitude R N up a straight road inclined at an angle α to the horizontal, working at the same power. Show that $v_1 = \dfrac{(R + Ma)v}{R + Mg \sin \alpha}$.



Let P N be the tractive force

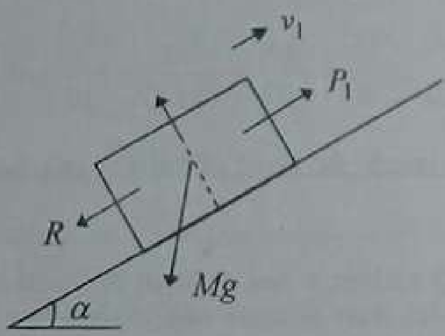
Apply $\underline{F} = m\underline{a} \rightarrow$:

$P - R = Ma$ -------------- (1) (5)

Let H W be the power of its engine.

Then $H = P \times v$ (5)

$= (R + Ma)v$ (by (1)) (5)



$\underline{F} = m\underline{a}$: ↗

$P_1 - R - Mg \sin \alpha = 0$ ---------- (2) (5)

Also $H = P_1 \times v_1$

$\therefore\ v_1 = \dfrac{H}{P_1} = \dfrac{(R + Ma)v}{(R + Mg \sin \alpha)}$. (by (2))

(5)

5. In the usual notation, let $\mathbf{a} = 3\mathbf{i} + 4\mathbf{j}$, $\mathbf{b} = 4\mathbf{i} + 3\mathbf{j}$ and $\mathbf{c} = \alpha\mathbf{i} + (1-\alpha)\mathbf{j}$, where $\alpha \in \mathbb{R}$.

Find (i) $|\mathbf{a}|$ and $|\mathbf{b}|$,

(ii) $\mathbf{a}\cdot\mathbf{c}$ and $\mathbf{b}\cdot\mathbf{c}$ in terms of α.

If the angle between $\mathbf{a}$ and $\mathbf{c}$ is equal to the angle between $\mathbf{b}$ and $\mathbf{c}$, show that $\alpha = \frac{1}{2}$.

(i) Magnitudes of vectors

$|\underline{a}| = \sqrt{3^2+4^2} = 5$

$|\underline{b}| = \sqrt{4^2+3^2} = 5$ } (5)

(5)

(ii) $\underline{a}\cdot\underline{c} = 3\alpha + 4(1-\alpha) = 4-\alpha$

(5)

$\underline{b}\cdot\underline{c} = 4\alpha + 3(1-\alpha) = 3+\alpha$

Let θ be the angle between $\underline{a}$ and $\underline{c}$. Then $\underline{a}\cdot\underline{c} = |\underline{a}||\underline{c}|\cos\theta$ and $\underline{b}\cdot\underline{c} = |\underline{b}||\underline{c}|\cos\theta$. (5)

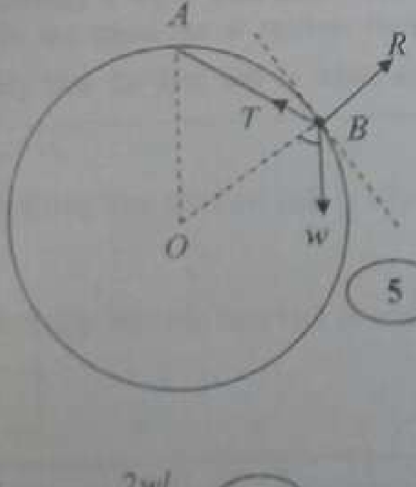
Since $|\underline{a}| = |\underline{b}|$, we have $\underline{a}\cdot\underline{c} = \underline{b}\cdot\underline{c}$.

$\therefore 4-\alpha = 3+\alpha$

$\Rightarrow \alpha = \frac{1}{2}$. (5)

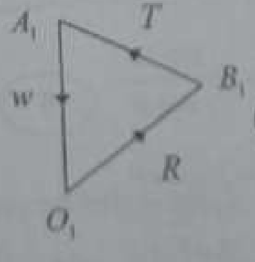
25

6. One end of a light inextensible string of length $2l$ is attached to the highest point of a thin smooth rigid circular wire of radius $a\left(>\sqrt{2}l\right)$ which is fixed in a vertical plane. A small smooth bead of weight w, which is free to move along the wire, is attached to the other end of the string. The bead is in equilibrium with the string taut, as in the figure. Mark the forces acting on the bead and show that the tension of the string is $\frac{2wl}{a}$.



(5)

Triangle of Forces



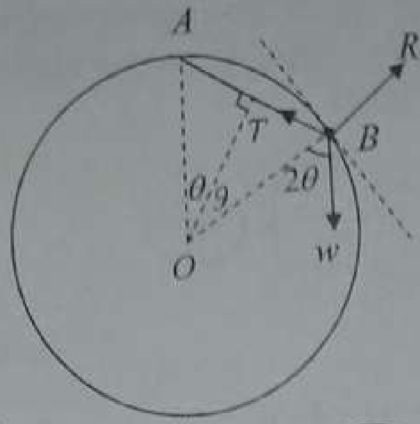
(10)

$\frac{T}{AB} = \frac{w}{OA} \Rightarrow T = \frac{2wl}{a}$ (5)

(5)

25

1



(5)

By Lami's Theorem, $\dfrac{T}{\sin(\pi - 2\theta)} = \dfrac{w}{\sin\left(\dfrac{\pi}{2} + \theta\right)}$. (10)

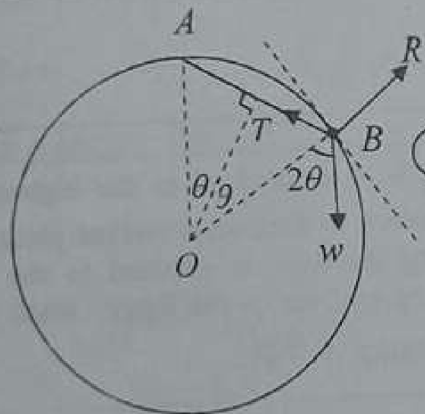
$\therefore \quad T = w\dfrac{\sin 2\theta}{\cos\theta}$

$= 2w\sin\theta = \dfrac{2wl}{a} \quad \left(\because \sin\theta = \dfrac{l}{a}\right)$. (5)

(5)

25

er 2



(5)

Resolve in a direction perpendicular to OB

$T\cos\theta = w\sin 2\theta$ (10)

$T = w\dfrac{\sin 2\theta}{\cos\theta}$ (5)

$= 2w\sin\theta$

$= \dfrac{2wl}{a} \quad \left(\because \sin\theta = \dfrac{l}{a}\right)$. (5)

25

7. Let A and B be two events of a sample space Ω. In the usual notation, $P(A) = p$, $P(B) = \frac{p}{2}$ and $P(A \cup B) - P(A \cap B) = \frac{2p}{3}$, where $p > 0$. Find $P(A \cap B)$ in terms of p.
Deduce that if A and B are independent events, then $p = \frac{5}{6}$.

For two events A and B, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. (5)

This gives us $P(A \cup B) = \frac{3p}{2} - P(A \cap B)$. ------------- (1) (5)

It is given that $P(A \cup B)) - P(A \cap B) = \frac{2p}{3}$. ------------- (2)

(1) and (2) $\Rightarrow \frac{3p}{2} - 2P(A \cap B) = \frac{2p}{3}$

$\Rightarrow P(A \cap B) = \frac{5p}{12}$. (5)

If A and B are independent, then $P(A \cap B) = P(A)\ P(B)$. (5)

$$\Rightarrow \frac{5p}{12} = p \cdot \frac{p}{2}$$

$$\Rightarrow p = \frac{5}{6}. \quad (\because p > 0)$$

(5)

8. A bag contains 6 white balls and n black balls which are equal in all respects, except for colo[ur]. Two balls are taken out at random from the bag, one after the other, without replacement. [The] probability that the first ball is white and the second ball is black is $\frac{4}{15}$. Find the value o[f n].

Probability that the first ball is white $= \frac{6}{n+6}$.

Probability that the first ball is white and the second ball is black $= \frac{6}{(n+6)} \cdot \frac{n}{(n+5)}$

$$\therefore \frac{6}{(n+6)} \cdot \frac{n}{(n+5)} = \frac{4}{15} \quad (5.)$$

$$\Rightarrow 2n^2 - 23n + 60 = 0 \quad (5.)$$

$\Rightarrow n = 4$. ($\because n$ is a positive integer) (5)

(Note: no marks if the answer is written as $n = 4$ or $n = 15/2$)

---

9. The mean of three distinct integers less than 11 is 7. When two more integers are taken, mean of all five integers is 5. Also, the only mode of these five integers is 3. Find the integers.

Let x, y, and z be distinct integers less than 11 with a mean of 7.

Then $\frac{x+y+z}{3} = 7$. (5)

$\Rightarrow x + y + z = 21$ ---------------(1)

Since x, y, and z are distinct and the only mode is 3, at least one of the two integers additionally taken must be 3. Let the other be t.

Since the mean of the five integers is 5, we have $\frac{x+y+z+t+3}{5} = 5$. (5)

$\Rightarrow \quad 21 + 3 + t = 25$

$\Rightarrow \quad t = 1$ (5)

Hence the integers are $x, y, z, 3, 1$. Since the only mode is 3, and x, y and z are distinct, exactly one of them must be 3. Let $z = 3$.

Again (1) $\Rightarrow x + y = 18$ ------------- (2) (5)

Since x and y are integers less than 11, (2) gives us

($x = 8$ and $y = 10$) or ($x = 10$ and $y = 8$). Hence, the five numbers are 1, 3, 3, 8 and 10.

(5) **25**

---

10. An arrow is shot at a rotating circular target-board consisting of five equal sectors numbered 1, 2, 3, 4 and 5. The number of times the arrow hits each of the sectors is given in the following frequency table, where p and q are constants.

Number	1	2	3	4	5
Frequency	1	p	q	5	2

If the mean and the variance of the above data are given to be 3 and $\frac{6}{5}$ respectively, find the values of p and q.

Mean $\mu = 3 \Rightarrow \frac{1 + 2p + 3q + 20 + 10}{p + q + 8} = 3$ (5)

$\Rightarrow 2p + 3q + 31 = 3p + 3q + 24$

$\Rightarrow \quad p = 7$. (5)

$$\sigma^2 = \frac{\sum_{i=1}^{n} f_i x_i^2}{\sum_{i=1}^{n} f_i} - \mu^2$$ (5)

Variance $= \frac{6}{5} \Rightarrow \frac{6}{5} = \frac{1 \cdot 1^2 + 7 \cdot 2^2 + q \cdot 3^2 + 5 \cdot 4^2 + 2 \cdot 5^2}{q + 15} - 3^2$ (5)

$\Rightarrow \quad 51(q + 15) = 5(1 + 28 + 9q + 80 + 50)$

$\Rightarrow \quad q = 5$. (5)

25

Aliter

$$\sigma^2 = \frac{\sum_{i=1}^{n} f_i (x_i - \bar{x})^2}{\sum_{i=1}^{n} f_i}$$ (5)

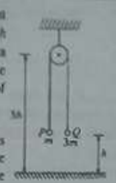
Variance $= \frac{6}{5} \Rightarrow \frac{6}{5} = \frac{1(1-3)^2 + 7(2-3)^2 + q(3-3)^2 + 5(4-3)^2 + 2(5-3)^2}{1 + 7 + q + 5 + 2}$ (5)

$\Rightarrow \quad \frac{6}{5} = \frac{4 + 7 + 5 + 8}{15 + q}$

$\Rightarrow \quad q = 5$. (5)

15

11. (a) A particle P of mass m is connected to a particle Q of mass $3m$ by a light inextensible string passing over a small smooth pulley fixed at a height $3h$ above an inelastic horizontal floor. Initially the two particles are held at a height h above the floor with the string taut, and released from rest. (See the adjoining figure.) Applying Newton's second law separately to the motions of P and Q, show that the magnitude of acceleration of each particle is $\frac{g}{2}$.

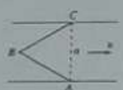


After a time t_0 the particle Q strikes the floor, comes to rest instantly, remains at rest for a further time t_1 and begins to move up. Sketch the velocity-time graphs separately for the motions of the two particles P and Q until the particle Q begins to move up.

Using these graphs, show that $t_0 = 2\sqrt{\frac{h}{g}}$ and find t_1 in terms of g and h.

Show further that the particle P reaches a maximum height $\frac{5h}{2}$ above the floor.

(b) A straight river of breadth a flows with uniform speed u. The points A and C are situated on opposite banks of the river such that the line AC is perpendicular to the direction of flow of the river. Also, a stationary buoy B is fixed in the middle of the river, on the upstream side of AC such that ABC is an equilateral triangle. (See the adjoining figure.)

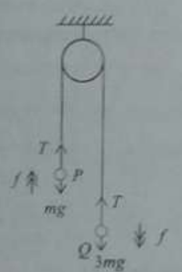


A boat moving with speed $v\ (> u)$ relative to water starts off from A and moves until it reaches B. Then it moves from B to C. Sketch the velocity triangles for the motions of the boat from A to B and from B to C.

Show that the speed of the boat in its motion from A to B is $\frac{1}{2}\left(\sqrt{4v^2 - u^2} - \sqrt{3}u\right)$ and find its speed in the motion from B to C.

Hence, show that the total time taken by the boat for the paths AB and BC is $\frac{a\sqrt{4v^2 - u^2}}{v^2 - u^2}$.

(a)



Apply $\underline{F} = m\underline{a}$:

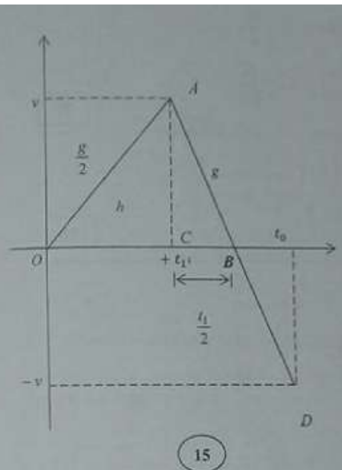
For $Q(3m)\downarrow$ $\quad 3mg - T = 3mf$ (5)

For $P(m)\uparrow$ $\quad T - mg = mf$ (5)

$2mg = 4mf$

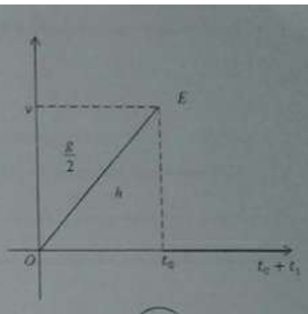
$\Rightarrow f = \frac{g}{2}.$ (5)

15



(15)

Particle P



(10)

Particle Q

25

From the v-t graphs

Area under OA or OE $\quad = \frac{1}{2} \cdot t_0 \cdot v = h$ ---------- (1) (5)

Gradient of OA or OE $\quad = \frac{v}{t_0} = \frac{g}{2}$ ---------- (2) (5)

$(1) \times (2) \Rightarrow \frac{1}{2} \cdot t_0 \cdot \frac{g t_0}{2} = h$

$\Rightarrow \quad t_0^2 = \frac{4h}{g}$

$\Rightarrow \quad t_0 = 2\sqrt{\frac{h}{g}}.$ (5)

15

Also (2) $\Rightarrow v = \frac{g}{2} \cdot 2\sqrt{\frac{h}{g}} = \sqrt{gh}$ (5)

For its motion under gravity alone, time taken by $P = \frac{2v}{g}$

$\therefore\ t_1 = 2\sqrt{\frac{h}{g}}$ (5)

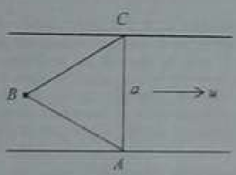
[10]

Maximum height reached by $P = \frac{1}{2} \cdot v \cdot \frac{t}{2} = \frac{1}{2}h$ (5)

Total height above the level of floor $= h + h + \frac{h}{2} = \frac{5h}{2}$ (5)

[10]

(b)



$\underline{V}(B,W) = v$

$\underline{V}(W,E) = u \ \rightarrow$

$\underline{V}(B,E) =$  for AB and  for BC (5) For both

$\underline{V}(B,E) = \underline{V}(B,W) + \underline{V}(W,E)$

$= \underline{V}(W,E) + \underline{V}(B,W)$

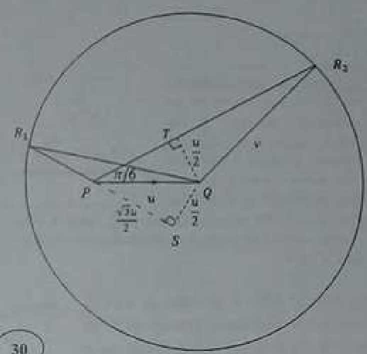
(5)

$= \overline{PQ} + \overline{QR_i}$

$= \overline{PR_i}$ for $i = 1, 2$, where $AB // PR_1$ and $BC // PR_2$ respectively.

(5) (5)

(Note: 10 marks each for setting up for each velocity triangle)



(30)

(Note: 15 marks each for each velocity triangle)

[30]

For AB: $\Delta\, PQR_1$

$R_1S = \sqrt{v^2 - \frac{u^2}{4}}$

$PR_1 = R_1S - PS$ (5)

$= \frac{1}{2}\left(\sqrt{4v^2 - u^2} - \sqrt{3}u\right).$ (5)

For BC: $\Delta\, PQR_2$

$PR_2 = PT + TR_2$

$= \frac{\sqrt{3}u}{2} + \sqrt{v^2 - \frac{u^2}{4}}$ (5)

$= \frac{1}{2}\left(\sqrt{4v^2 - u^2} + \sqrt{3}u\right)$ (5)

$PR_1 . PR_2 = \frac{1}{4}\left(4v^2 - u^2 - 3u^2\right)$

$PR_1 + PR_2 = \sqrt{4v^2 - u^2}$
$PR_1 \cdot PR_2 = v^2 - u^2$

Total Time $= \frac{a}{PR_1} + \frac{a}{PR_2}$

$= a\,\frac{(PR_1 + PR_2)}{PR_1 . PR_2}$

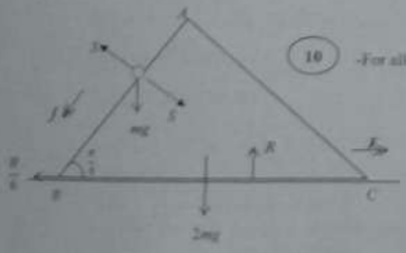
$= a\,\frac{\sqrt{4v^2 - u^2}}{\ldots}$ (5)

[25]

12 (a) The triangle ABC in the figure is a vertical cross-section through the centre of gravity of a uniform wedge of mass $2m$. The line AB is a line of greatest slope of the face containing it and $A\hat{B}C = \frac{\pi}{4}$. The wedge is placed with the face containing BC on a rough horizontal floor. The face containing AB is smooth. A particle of mass m is held on AB as in the figure and the system is released from rest. It is given that the wedge moves in the direction of $\overrightarrow{BC}$ and that the magnitude of the frictional force exerted on the wedge by the floor is $\frac{R}{6}$, where R is the magnitude of the normal reaction exerted on the wedge by the floor. Obtain equations which are sufficient to determine R, in terms of m and g.

(b) OAB in the figure is a circular sector of radius a subtending an angle $\frac{\pi}{6}$ at the centre O with OA vertical. It is a cross-section perpendicular to the axis of a smooth cylindrical sector fixed with its axis horizontal. One end of a light inextensible string passing over a small smooth pulley fixed at B is attached to a particle P of mass $3m$ and the other end is attached to a particle Q of mass m. Initially, the particle P is held at A and the particle Q hangs freely at the horizontal level of O. The system is released from rest in this position, with the string taut. When OP makes an angle $\theta\left(0 < \theta < \frac{\pi}{6}\right)$ with the upward vertical, show that $2a\dot{\theta}^2 = 3g(1-\cos\theta) + g\theta$ and that the tension in the string is $\frac{3}{4}mg(1-\sin\theta)$, and find the normal reaction on the particle P.

(a)

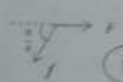


(10) -For all forces

$\underline{a}(2m, E) = F \longrightarrow$

$\underline{a}(m, 2m) = f \swarrow$

$\underline{a}(m, E) = \underline{a}(m, 2m) + \underline{a}(2m, E)$



(10)

Apply $\underline{F} = m\underline{a}$.

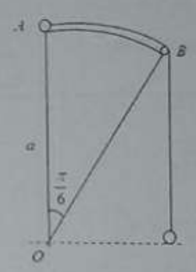
(i) For the Particle $\swarrow mg\frac{\sqrt{2}}{2} = m\left(f - F\left(\frac{\sqrt{2}}{2}\right)\right)$ (15)

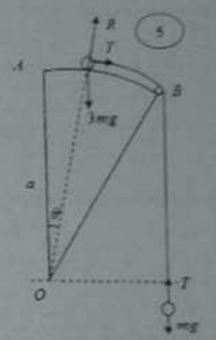
(ii) For the System $\rightarrow \frac{-R}{6} = 2mF + m\left(F - \frac{f}{\sqrt{2}}\right)$ (15)

(iii) For the system $\uparrow$ $R - 3mg = -m\frac{f}{\sqrt{2}}$ (10)

60

(b)





(5) (5)

By Conservation of Mechanical Energy

$3mga = 3mga\cos\theta - mga\theta + \frac{1}{2}(3m)(a\dot{\theta})^2 + \frac{1}{2}(m)(a\dot{\theta})^2$ (25) { PE 10, KE10, Equation 05 }

$2a\dot{\theta}^2 = 3g(1-\cos\theta) + g\theta$ (5)

40

Applying $\underline{F} = m\underline{a}$

For P $\searrow T + 3mg\sin\theta = 3mf$ ------ (1) (15)

For Q $\downarrow mg - T = mf$ ------ (2) (10)

By (1) and (2),

$$3mg - 3T = T + 3mg\sin\theta$$
$$4T = 3mg(1 - \sin\theta) \quad (5)$$
$$T = \frac{3mg}{4}(1 - \sin\theta)$$

[30]

Applying $\underline{F} = m\underline{a}$ for P

$\swarrow$ $3mg\cos\theta - R = 3ma\dot{\theta}^2$ (10)

$R = 3mg\cos\theta - \frac{3m}{2}\{3g(1 - \cos\theta) + g\theta\}$ (10)

$$= \frac{3mg}{2}(2\cos\theta - 3 + 3\cos\theta - \theta)$$
$$= \frac{3mg}{2}(5\cos\theta - \theta - 3)$$

[20]

Note:

P will not leave the surface for $0 < \theta < \frac{\pi}{6}$

$$R|_{\theta=0} = 3mg > 0$$
$$\frac{dR}{d\theta} = \frac{3mg}{2}(-5\sin\theta - 1) < 0 \; for \; 0 < \theta < \frac{\pi}{6}$$
$$R|_{\theta=\frac{\pi}{6}} = \frac{3mg}{2}\left(\frac{5\sqrt{3}}{2} - \frac{\pi}{6} - 3\right) > 0$$

13. One end of a light elastic string of natural length a and modulus of elasticity $4mg$ is tied to a fixed point O and the other end to a particle P of mass m. The particle P is released from rest at O. Find the velocity of the particle P when is passes through the point A, where $OA = a$.

Show that the length of the string $x (\geq a)$ satisfies the equation $\ddot{x} + \frac{4g}{a}\left(x - \frac{5a}{4}\right) = 0$.

Taking $X = x - \frac{5a}{4}$, express the above equation in the form $\ddot{X} + \omega^2 X = 0$, where $\omega (> 0)$ is a constant to be determined.

Assuming that $\dot{X}^2 = \omega^2\left(c^2 - X^2\right)$, find the amplitude c of this simple harmonic motion.

Let L be the lowest point reached by the particle P. Show that the time taken by P to move from A to L is $\frac{1}{2}\sqrt{\frac{a}{g}}\left\{\pi - \cos^{-1}\left(\frac{1}{3}\right)\right\}$.

At the instant when the particle P is at L another particle of mass λm $(1 \leq \lambda < 3)$ is gently attached to P. Show that the equation of motion of the composite particle of mass $(1 + \lambda)m$ is $\ddot{x} + \frac{4g}{(1 + \lambda)a}\left\{x - (5 + \lambda)\frac{a}{4}\right\} = 0$.

Show further that the composite particle performs complete simple harmonic motion with amplitude $(3 - \lambda)\frac{a}{4}$

For motion of P under gravity alone

from O to A :

$\downarrow$ $v^2 = 2ga \Rightarrow v = \sqrt{2ag}$. (05)

Tension in the string: $T = \frac{4mg(x - a)}{a}$, $x \geq a$ (5)

$\downarrow$ $\underline{F} = m\underline{a}$: $-T + mg = m\ddot{x}$ (05)

Eliminating T: $-4mg\frac{(x - a)}{a} + mg = m\ddot{x}$ (5)

$$\Rightarrow \ddot{x} + \frac{4g}{a}(x - a) = \frac{4g}{a}\cdot\frac{a}{4}$$
$$\Rightarrow \ddot{x} + \frac{4g}{a}\left(x - \frac{5a}{4}\right) = 0 \text{ ---------- (1)} \quad (5)$$

Write $X = x - \frac{5a}{4} \Rightarrow \dot{X} = \dot{x}$ and $\ddot{X} = \ddot{x}$. (5)

Then (1) becomes $\ddot{X} + \frac{4g}{a}X = 0$

Hence $\ddot{X} + \omega^2 X = 0$, where $\omega = 2\sqrt{\frac{g}{a}}$. $(\because \omega > 0)$

(5) (5)

[40]

$\Rightarrow \dot{X}^2 = \omega^2(c^2 - X^2)$ ---------- (2)

$\dot{x} = \sqrt{2ga}$ when $x = a$ $\Rightarrow \dot{X}^2 = 2ga$ when $X = -\frac{a}{4}$.

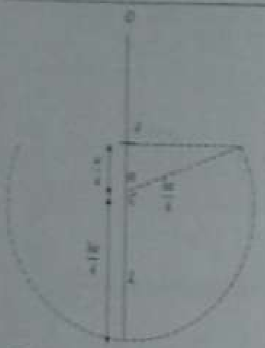
(5) (5)

Then (2) $\Rightarrow 2ga = \frac{4g}{a}\left[c^2 - \left(\frac{-a}{4}\right)^2\right]$ (5)

$\Rightarrow c = \frac{3a}{4} \quad (\because c > 0)$ (5)

Centre is given by $\ddot{X} = 0$; i.e $x = \frac{5a}{4}$ (5)

25



$AL = \frac{a}{4} + \frac{3a}{4} = a$ (5)

$\cos\alpha = \frac{1}{3}$ (5)

Time taken from A to $L = \frac{\pi - \alpha}{\omega}$ (5)

$= \frac{1}{2}\sqrt{\frac{a}{g}}\left\{\pi - \cos^{-1}\left(\frac{1}{3}\right)\right\}$ (5)

(15)

(With the circular arc, angle and distances marked)

35

$T_1 = \frac{4mg(x-a)}{a}$

For composite particle $\underline{F} = m\underline{a}$:

$(1+\lambda)mg - T_1 = (1+\lambda)m\ddot{x}$ (10)

$(1+\lambda)mg - \frac{4mg}{a}(x-a) = (1+\lambda)m\ddot{x}$ (5)

$\ddot{x} + \frac{4g}{(1+\lambda)a}(x-a) - g = 0$ (5)

$\ddot{x} + \frac{4g}{(1+\lambda)a}\left\{x - a - (1+\lambda)\frac{a}{4}\right\} = 0$

$\ddot{x} + \frac{4g}{(1-\lambda)a}\left\{x - (5+\lambda)\frac{a}{4}\right\} = 0$ (5)

25

Centre C_1: $x = OC_1 = (5+\lambda)\frac{a}{4}$

$C_1L = 2a - (5+\lambda)\frac{a}{4}$ (5)

$= (3-\lambda)\frac{a}{4}$ (5)

New amplitude $c_1 = (3-\lambda)\frac{a}{4} \quad (>0) \because \lambda < 1$

Complete Simple Harmonic Motion if and only if

$AC_1 \geq c_1$ (5)

$(5+\lambda)\frac{a}{4} - a \geq (3-\lambda)\frac{a}{4}$

$5 + \lambda - 4 \geq 3 - \lambda$

$\lambda \geq 1$ (5) **25**

Aliter

Let $X = A\cos\omega t + B\sin\omega t$, where A and B are constants to be determined.

$\Rightarrow \dot{X} = -A\omega\sin\omega t + B\omega\cos\omega t$. (5)

When $t = 0$ and $x = a$, $X = -\frac{a}{4}$ and $\dot{X} = V = \sqrt{2ga}$

(5) (5)

$\therefore -\frac{a}{4} = A$ and $V = B\omega \Rightarrow B = \frac{V}{\omega}$

(5) (5)

Solution: $X = -\frac{a}{4}\cos\omega t + \frac{V}{\omega}\sin\omega t$. **25**

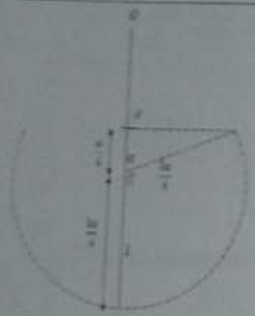
Differentiating: $\dot{X} = \frac{a\omega}{4}\sin\omega t + V\cos\omega t$ (5)

Lowest point L is reached when $\dot{X} = 0$ (5)

$\Rightarrow c = \dfrac{3a}{4}$ $(\because c > 0)$ (5)

Centre is given by $\ddot{X} = 0$; i.e. $x = \dfrac{5a}{4}$ (5)

25



$AL = \dfrac{a}{4} + \dfrac{3a}{4} = a$ (5)

$\cos\alpha = \dfrac{1}{3}$ (5)

Time taken from A to $L = \dfrac{\pi - \alpha}{\omega}$ (5)

$= \dfrac{1}{2}\sqrt{\dfrac{a}{g}}\left\{\pi - \cos^{-1}\left(\dfrac{1}{3}\right)\right\}$ (5)

(15)

(With the circular arc, angle and distances marked)

35

$T_1 = \dfrac{4mg(x-a)}{a}$

For composite particle $F = m\ddot{x}$:

$(1+\lambda)mg - T_1 = (1+\lambda)m\ddot{x}$ (10)

$(1+\lambda)mg - \dfrac{4mg}{a}(x-a) = (1+\lambda)m\ddot{x}$ (5)

$\ddot{x} + \dfrac{4g}{(1+\lambda)a}(x-a) - g = 0$ (5)

$\ddot{x} + \dfrac{4g}{(1+\lambda)a}\left\{x - a - (1+\lambda)\dfrac{a}{4}\right\} = 0$

$\ddot{x} + \dfrac{4g}{(1+\lambda)a}\left\{x - (5+\lambda)\dfrac{a}{4}\right\} = 0$ (5)

25

Centre C_1 : $x = OC_1 = (5+\lambda)\dfrac{a}{4}$

$C_1L = 2a - (5+\lambda)\dfrac{a}{4}$ (5)

$= (3-\lambda)\dfrac{a}{4}$ (5)

New amplitude $c_1 = (3-\lambda)\dfrac{a}{4}$ (>0) $\because \lambda < 3$

Complete Simple Harmonic Motion if and only if

$AC_1 \geq c_1$ (5)

$(5+\lambda)\dfrac{a}{4} - a \geq (3-\lambda)\dfrac{a}{4}$

$5 + \lambda - 4 \geq 3 - \lambda$

$\lambda \geq 1$ (5) 25

Aliter

Let $X = A\cos\omega t + B\sin\omega t$, where A and B are constants to be determined.

$\Rightarrow \dot{X} = -A\omega\sin\omega t + B\omega\cos\omega t$. (5)

When $t = 0$ and $x = a$, $X = -\dfrac{a}{4}$ and $\dot{X} = V = \sqrt{2ga}$

(5) (5)

$\therefore -\dfrac{a}{4} = A$ and $V = B\omega \Rightarrow B = \dfrac{V}{\omega}$

(5) (5)

Solution: $X = -\dfrac{a}{4}\cos\omega t + \dfrac{V}{\omega}\sin\omega t$.

25

Differentiating: $\dot{X} = \dfrac{a\omega}{4}\sin\omega t + V\cos\omega t$ (5)

(5)

Lowest point L is reached when $\dot{X} = 0$

Then $\tan \omega t_1 = -\frac{4V}{a\omega}$ (5)

$\omega t_1 = \pi - \alpha$

$\tan \alpha = \frac{4V}{a\omega}$ where $0 < \alpha < \frac{\pi}{2}$ (5)

Centre C of SHS is such that $x = \frac{5a}{4}$ or $AC = \frac{a}{4}$ (5)

$$\frac{a}{4} = c\cos\alpha = \frac{c(a\omega)}{\sqrt{16V^2 + a^2\omega^2}}$$

$$= c\,\frac{2\sqrt{ga}}{\sqrt{16\times 2ga + 4ga}} = \frac{1}{3}c$$ (5)

$$\Rightarrow c = \frac{3a}{4}$$

Also from above $\omega t_1 = \pi - \cos^{-1}\left(\frac{1}{3}\right)$ (5)

$$t_1 = \frac{1}{\omega}\left\{\pi - \cos^{-1}\left(\frac{1}{3}\right)\right\}.$$

35

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14. (a) The position vectors of two points A and B with respect to an origin O are **a** and **b** respectively, where O, A and B are **not collinear**. Let C be the point such that $\overrightarrow{OC} = \frac{1}{3}\overrightarrow{OB}$ and let D be the point such that $\overrightarrow{OD} = \frac{1}{2}\overrightarrow{AB}$. By expressing $\overrightarrow{AC}$ and $\overrightarrow{AD}$ in terms of **a** and **b**, show that $\overrightarrow{AD} = \frac{3}{2}\overrightarrow{AC}$.

Let P and Q be the points on AB and OD respectively, such that $\overrightarrow{AP} = \lambda\overrightarrow{AB}$ and $\overrightarrow{OQ} = (1-\lambda)\overrightarrow{OD}$, where $0 < \lambda < 1$. Show that $\overrightarrow{PC} = 2\overrightarrow{CQ}$.

(b) In a parallelogram $ABCD$, let $AB = 2$ m and $AD = 1$ m, and let $B\hat{A}D = \frac{\pi}{3}$. Also, let E be the mid-point of CD. Forces of magnitudes 5, 5, 2, 4 and 3 newtons act along AB, BC, DC, DA and BE respectively, in the directions indicated by order of the letters. Show that their resultant force is parallel to $\overrightarrow{AE}$, and find its magnitude.

Also, show that the line of action of the resultant force meets AB produced at a distance $\frac{3}{2}$ m from B.

An additional force acting through C is now added to the above system of forces so that the resultant force of the new system is along $\overrightarrow{AE}$. Find the magnitude and direction of the additional force.

Let $\overrightarrow{OA} = \underline{a}$ and $\overrightarrow{OB} = \underline{b}$

Then $\overrightarrow{OC} = \frac{1}{3}\overrightarrow{OB} = \frac{\underline{b}}{3}$ (5) and $\overrightarrow{OD} = \frac{1}{2}\overrightarrow{AB} = \frac{\underline{b}-\underline{a}}{2}$ (5)

$\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{OD}$

$= -\underline{a} + \frac{\underline{b}}{2} - \frac{\underline{a}}{2}$

$= \frac{3}{2}\left(-\underline{a} + \frac{\underline{b}}{3}\right)$ ---------- (1) (5)

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$

$= -\underline{a} + \frac{\underline{b}}{3}$ ---------- (2) (5)

By (1) and (2), $\overrightarrow{AD} = \frac{3}{2}\overrightarrow{AC}$ (5)

25

$\overrightarrow{PC} = \overrightarrow{PO} + \overrightarrow{OC}$

$= \overrightarrow{PA} + \overrightarrow{AO} + \overrightarrow{OC}$

$= -\lambda\overrightarrow{AB} - \overrightarrow{OA} + \overrightarrow{OC}$ (5)

$= -\lambda(\underline{b} - \underline{a}) - \underline{a} + \underline{c}$ (5)

$= (\lambda - 1)\underline{a} - \lambda\underline{b} + \frac{\underline{b}}{3}$

$= (\lambda - 1)\underline{a} + \frac{1}{3}(1 - 3\lambda)\underline{b}$ ------- (3) (5)

$\overrightarrow{CQ} = \overrightarrow{CO} + \overrightarrow{OQ}$

$= -\overrightarrow{OC} + (1-\lambda)\overrightarrow{OD}$ (5)

$= -\frac{\underline{b}}{3} + (1-\lambda)\frac{1}{2}(\underline{b} - \underline{a})$ (5)

$= \frac{1}{2}\left[(\lambda - 1)\underline{a} - \frac{3}{2}\underline{b} + \underline{b} - \lambda\underline{b}\right]$

$= \frac{1}{2}\left((\lambda - 1)\underline{a} + \frac{1}{3}(1 - 3\lambda)\underline{b}\right)$ ------- (4) (5)

By (3) and (4), $\overrightarrow{PC} = 2\overrightarrow{CQ}$ (5)

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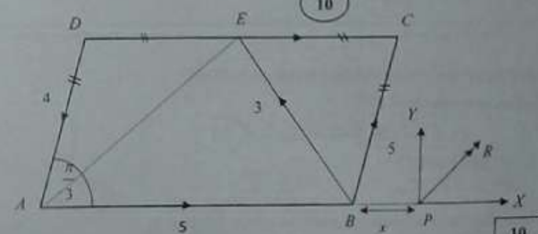
(b) (10) -For marking the given forces



10

Resolving parallel to $\overline{AE}$: $= -4\cos\frac{\pi}{6} + 2\cos\frac{\pi}{6} + 5\cos\frac{\pi}{6} + 5\cos\frac{\pi}{6}$ (10)

$= 4\sqrt{3}N$ (5)

Resolving (perpendicular to AE) : $3 - 4\sin\frac{\pi}{6} + 5\sin\frac{\pi}{6} - 5\sin\frac{\pi}{6} - 2\cos\frac{\pi}{6}$ (10)

$= 3 - 2 + \frac{5}{2} - \frac{5}{2} - 1$

$= 0$ (5)

Resultant R is of magnitude $4\sqrt{3}N$; // to $\overline{AE}$.

(5) (5)

40

OR $\rightarrow X = 5 + \frac{3}{2} + 2 - \frac{3}{2} - \frac{4}{2} = 6N$ (10)

$\uparrow Y = \frac{\sqrt{3}}{2}(5+3) - \frac{4\sqrt{3}}{2} = 2\sqrt{3}N$ (10)

$\frac{Y}{X} = \frac{2\sqrt{3}}{6} = \frac{1}{\sqrt{3}}$. (5)

Resultant R is of magnitude $2\sqrt{3}\sqrt{3+1} = 4\sqrt{3}N$, (5)

Its line of action makes an angle,

$\tan^{-1}(1/\sqrt{3}) = \pi/6$, with side AB. $\therefore$ it is parallel to AE.

(5) (5) 40

Let P be the point where resultant meets AB produced.

Taking moments about B ↻

$Yx = 4 \times 2\sin\frac{\pi}{3} - 2 \times 1\sin\frac{\pi}{3}$ (10)

$2\sqrt{3}x = 3\sqrt{3}$

$x = \frac{3}{2}$ m. (5)

Line of action of R meets AB produced at $x = \frac{3}{2}$ m from B.

15

Additional force must be parallel to $\overline{EA}$. (5)

A ↺ $R \times \left(2 + \frac{3}{2}\right)\sin 30^{\circ} = F.1\sin 30^{\circ}$ (15)

$4\sqrt{3} \times \frac{7}{2} = F$

$F = 14\sqrt{3}N$. (5)

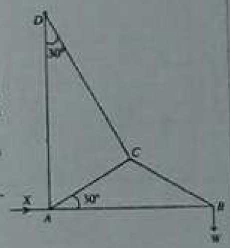
25

15 (a) Four equal uniform rods, each of weight w_1, are smoothly jointed at their ends to form a rhombus $ABCD$. The mid-points of BC and CD are connected by a light rod such that $B\hat{A}D = 2\theta$. Each of the joints B and D carries equal loads of weight w_2. The system, hanging symmetrically from the joint A, is in equilibrium in a vertical plane with the light rod horizontal. Show that the thrust in the light rod is $2(2w_1 + w_2)\tan\theta$.

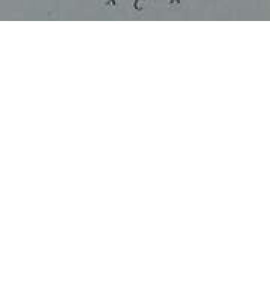
(b) The adjoining figure represents a framework of five light rods AB, BC, CD, AC and AD, smoothly jointed at the ends. It is given that $AC = CB$ and $B\hat{A}C = 30^{\circ} = A\hat{D}C$. The framework is smoothly hinged at D. A weight W is suspended at the joint B and the framework is kept in equilibrium in a vertical plane with AB horizontal and AD vertical, by a horizontal force of magnitude X acting at A.

Using Bow's notation, draw stress diagrams for the joints B, C and A in the same figure.

Hence, find the value of X and the stresses in all rods, distinguishing between tensions and thrusts.



(a)



(10)

Symmetry (5)

Let $2a$ be the length of each heavy rod.

$\overset{\curvearrowleft}{B}$ for BC : $X \times 2a\cos\theta - w_1 \times a\sin\theta - T \times a\cos\theta = 0$. (10)

$\Rightarrow 2X - T = w_1 \tan\theta$ ---------- (1) (5)

$\overset{\curvearrowleft}{A}$ for AB and BC :

$X \times 4a\cos\theta + 2w_1 \times a\sin\theta + w_2 \times 2a\sin\theta - T \times 3a\cos\theta = 0$. (20)

$\Rightarrow 4X - 3T = -2(w_1 + w_2)\tan\theta$ ---------- (2) (5)

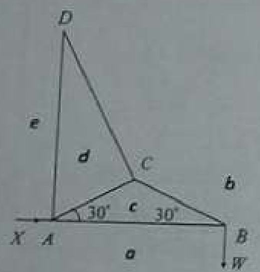
(1) $\times 2$ - (2) $\Rightarrow T = 2w_1\tan\theta + (2w_1 + w_2)\tan\theta$

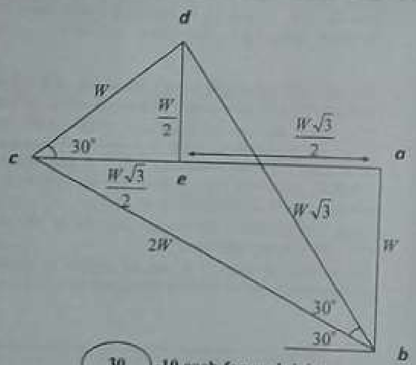
$= (4w_1 + 2w_2)\tan\theta$

$= 2(2w_1 + w_2)\tan\theta$ (10)

65

(b)





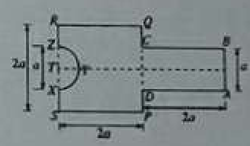
(30) -10 each for each joint

Rod	Magnitude	Tension/Trust	
BC(bc)	$2W$ (5)	Tension	(5)
AB(ca)	$\sqrt{3}W$ (5)	Thrust	(5)
CD(bd)	$\sqrt{3}W$ (5)	Tension	(5)
AC(dc)	W (5)	Tension	(5)
AD(de)	$\frac{W}{2}$ (5)	Thrust	(5)
$X(ea) = \frac{W\sqrt{3}}{2}$			(5)

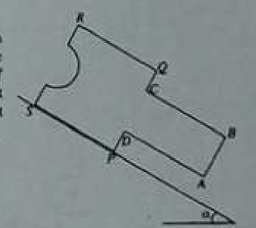
85

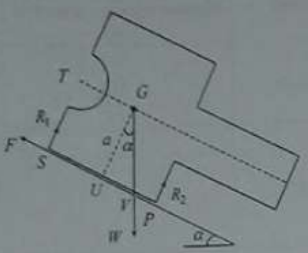
16. Show that the centre of mass of a uniform semi-circular lamina of radius r and centre O is at a distance $\frac{4r}{3\pi}$ from O.

As shown in the adjoining figure, a uniform plane lamina L is made by rigidly attaching a rectangle $ABCD$ to a square $PQRS$ such that DC and PQ lie on the same line with their mid-points coinciding, and removing a semi-circular region XYZ of radius $\frac{a}{2}$ centred at the mid-point T of RS. It is given that $AB = a$ and $AD = PQ = 2a$. Show that the centre of mass of the lamina L lies on the axis of symmetry at a distance ka from RS, where $k = \frac{238}{3(48-\pi)}$.



As shown in the adjoining figure, the lamina L is in equilibrium on a rough plane inclined at an angle α to the horizontal with its plane vertical and the edge PS on a line of greatest slope such that the point P lies below S. Show that $\tan\alpha < (2-k)$ and $\mu \geq \tan\alpha$, where μ is the coefficient of friction between the lamina and the inclined plane.





(10)

(10)

Since, PS is in contact with the plane, we must have $UV < UP$.

i.e. $a \tan \alpha < 2a - ka$

$\Rightarrow \tan \alpha < (2 - k)$. (Note that $k < 2$.)

(10)

$R_1 + R_2 = w \cos \alpha$ (10)

$F = w \sin \alpha$ (5)

Since L does not slide, we must have $\mu \geq \dfrac{F}{R_1 + R_2}$. (10)

This gives us $\mu \geq \tan \alpha$. (5)

60

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17 (a) An unbiased cubical die A shows 1, 2, 3, 3, 4, 5 on its six separate faces. The die A is tossed twice. Find the probability that the sum of the two numbers obtained is 6.

Another die B, identical to A in all respects except for the numbers on the faces, shows 2, 2, 3, 4, 4, 5 on its six separate faces. The die B is tossed twice. Find the probability that the sum of the two numbers obtained is 6.

Now, the two dice A and B are put in a box. One die is taken out of the box at random and tossed twice. Given that the sum of the two numbers obtained is 6, find the probability that the die taken out of the box is the die A.

(b) The mean and the standard deviation of n numbers $x_1, x_2, \dots, x_n$ are μ_1 and σ_1 respectively, and the mean and the standard deviation of m numbers $y_1, y_2, \dots, y_m$ are μ_2 and σ_2 respectively. Let the mean and the standard deviation of all of these $n + m$ numbers be μ_3 and σ_3 respectively.

Show that $\mu_3 = \dfrac{n\mu_1 + m\mu_2}{n + m}$.

Let $d_1 = \mu_3 - \mu_1$. Show that $\sum_{i=1}^{n} (x_i - \mu_3)^2 = n(\sigma_1^2 + d_1^2)$.

By taking $d_2 = \mu_3 - \mu_2$, write down a similar expression for $\sum_{j=1}^{m} (y_j - \mu_3)^2$.

Deduce that $\sigma_3^2 = \dfrac{(n\sigma_1^2 + m\sigma_2^2) + (nd_1^2 + md_2^2)}{n + m}$.

The number of copies sold per day, during the first 100 days after publishing a new book, had mean 2.3 and variance 0.8. During the next 100 days, the number of copies sold per day had mean 1.7 and variance 0.5. Find the mean and the variance of the number of copies sold per day during the first 200 days.

(a) In a single toss of the die A, the probability $P(n)$ of obtaining a face with number n is given below:

n	1	2	3	4	5
$P(n)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$

Let X_i be the number obtained in the i^{th} toss for $i = 1, 2$.

Then $P(X_1 + X_2 = 6) = P(X_1 = 1 \text{ and } X_2 = 5) + P(X_1 = 5 \text{ and } X_2 = 1)$

$+ P(X_1 = 2 \text{ and } X_2 = 4) + P(X_1 = 4 \text{ and } X_2 = 2)$

$+ P(X_1 = 3 \text{ and } X_2 = 3)$.

$= 4 \times \frac{1}{6} \times \frac{1}{6} + \frac{1}{3} \times \frac{1}{3}$ (15)

$= \frac{2}{9}$. (5)

20

For the die B, replace X by Y.

x	2	3	4	5
$P(x)$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$

Then $P(Y_1 + Y_2 = 6) = 2 \times \frac{1}{3} \times \frac{1}{3} + \frac{1}{6} \times \frac{1}{6}$ (15)

$= \frac{1}{4}$ (5)

[20]

By Bayes' Theorem,

$$P(A|\text{sum}=6) = \frac{P(\text{sum}=6|A)P(A)}{P(\text{sum}=6|A)P(A) + P(\text{sum}=6|B)P(B)}$$ (10)

(5)

$$= \frac{\frac{2}{9} \times \frac{1}{2}}{\frac{2}{9} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{2}} = \frac{8}{17}$$ (5)

(10)

[30]

(b) $\mu_1 = \frac{\sum_{i=1}^{n} x_i}{n}$ $\mu_2 = \frac{\sum_{i=1}^{m} y_i}{m}$ (5) -For both

Now $\mu_3 = \frac{\sum_{i=1}^{n} x_i + \sum_{i=1}^{m} y_i}{m+n}$ (5)

$= \frac{n\mu_1 + m\mu_2}{m+n}$ (5)

[15]

$$\sum_{i=1}^{n}(x_i - \mu_3)^2 = \sum_{i=1}^{n}(x_i - \mu_1 + \mu_1 - \mu_3)^2$$ (5)

$$= \sum_{i=1}^{n}(x_i - \mu_1 - d_1)^2$$

$$= \sum_{i=1}^{n}\left[(x_i - \mu_1)^2 - 2d_1(x_i - \mu_1) + d_1^2\right]$$ (5)

$$= \sum_{i=1}^{n}(x_i - \mu_1)^2 - 2d_1\sum_{i=1}^{n}(x_i - \mu_1) + \sum_{i=1}^{n} d_1^2$$ (5)

$$= n\sigma_1^2 + nd_1^2 \quad \left(\because \sum_{i=1}^{n}(x_i - \mu_1) = 0 \text{ and } \sigma_1^2 = \frac{\sum_{i=1}^{n}(x_i - \mu_1)^2}{n}\right)$$ (5) (5)

$$= n(\sigma_1^2 + d_1^2)$$ (5)

[30]

Similarly, $\sum_{j=1}^{m}(y_j - \mu_3)^2 = m(\sigma_2^2 + d_2^2)$, where $d_2 = \mu_3 - \mu_2$. (5) [05]

$$\sigma_3^2 = \frac{\sum_{i=1}^{n}(x_i - \mu_3)^2 + \sum_{j=1}^{m}(y_j - \mu_3)^2}{m+n}$$ (5)

$$= \frac{n(\sigma_1^2 + d_1^2) + m(\sigma_2^2 + d_2^2)}{m+n}$$

$$= \frac{(n\sigma_1^2 + m\sigma_2^2) + n(d_1^2 + md_2^2)}{m+n}$$ (5)

[10]

For the first 100 :

$n = 100$, $\mu_1 = 2.3$, $\sigma_1 = 0.8$

For the second 100 :

$m = 100$, $\mu_2 = 1.7$, $\sigma_2 = 0.5$

(5)

From the above, $\mu_3 = \frac{230 + 170}{200} = 2$ (5)

Note that $d_1 = -0.3$ and $d_2 = 0.3$.

$$\sigma_3^2 = \frac{100}{200}\left(0.8^2 + 0.5^2 + (0.3)^2 + (0.3)^2\right)$$ (5)

For the die B, replace X by Y.

y	2	3	4	5
$P(y)$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$

Then $P(Y_1 + Y_2 = 6) = 2 \times \frac{1}{3} \times \frac{1}{3} + \frac{1}{6} \times \frac{1}{6}$ (15)

$= \frac{1}{4}$ (5)

[20]

By Bayes' Theorem,

$$P(A \mid \text{sum} = 6) = \frac{P(\text{sum} = 6 \mid A)P(A)}{P(\text{sum} = 6 \mid A)P(A) + P(\text{sum} = 6 \mid B)P(B)}$$ (10)

(5)

$$= \frac{\frac{2}{9} \times \frac{1}{2}}{\frac{2}{9} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{2}} = \frac{8}{17}$$ (5)

(10)

[30]

(ii) $\mu_1 = \frac{\sum_{i=1}^{n} x_i}{n}$ $\qquad \mu_2 = \frac{\sum_{j=1}^{m} y_j}{m}$ (5) For both

Now $\mu_3 = \frac{\sum_{i=1}^{n} x_i + \sum_{j=1}^{m} y_j}{m + n}$ (5)

$= \frac{n\mu_1 + m\mu_2}{m + n}$ (5)

[15]

$$\sum_{i=1}^{n}(x_i - \mu_3)^2 = \sum_{i=1}^{n}(x_i - \mu_1 + \mu_1 - \mu_3)^2$$ (5)

$$= \sum_{i=1}^{n}(x_i - \mu_1 - d_1)^2$$

$$= \sum_{i=1}^{n}\left\{(x_i - \mu_1)^2 + 2d_1(x_i - \mu_1) + d_1^2\right\}$$ (5)

$$= \sum_{i=1}^{n}(x_i - \mu_1)^2 - 2d_1\sum_{i=1}^{n}(x_i - \mu_1) + \sum_{i=1}^{n} d_1^2$$ (5)

$$= n\sigma_1^2 + nd_1^2 \quad \left(\because \sum_{i=1}^{n}(x_i - \mu_1) = 0 \text{ and } \sigma_1^2 = \frac{\sum_{i=1}^{n}(x_i - \mu_1)^2}{n}\right)$$ (5) (5)

$$= n(\sigma_1^2 + d_1^2)$$ (5)

[30]

Similarly, $\sum_{j=1}^{m}(y_j - \mu_3)^2 = m(\sigma_2^2 + d_2^2)$, where $d_2 = \mu_3 - \mu_2$ (5)

[05]

$$\sigma_3^2 = \frac{\sum_{i=1}^{n}(x_i - \mu_3)^2 + \sum_{j=1}^{m}(y_j - \mu_3)^2}{m + n}$$ (5)

$$= \frac{n(\sigma_1^2 + d_1^2) + m(\sigma_2^2 + d_2^2)}{m + n}$$

$$= \frac{(n\sigma_1^2 + m\sigma_2^2) + n(d_1^2 + md_2^2)}{m + n}$$ (5)

[10]

For the first 100 :

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(5)

From the above, $\mu_3 = \frac{230 + 170}{200} = 2$. (5)

Note that $d_1 = -0.3$, and $d_2 = 0.3$.

$$\sigma_3^2 = \frac{100}{200}\left(0.8^2 + 0.5^2 + (0.3)^2 + (0.3)^2\right)$$ (5)

$$= \frac{1}{2}[0.64 + 0.25 + 0.09 \times 2]$$

$$= \frac{1.07}{2} = 0.535$$

$\sigma_3 = \sqrt{0.535}$. (5)

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